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DE MORGAN AS LOGICIAN.

BY GEORGE BRUCE HALSTED.

Augustus De Morgan was born, in 1806, in India, where his father was in the East India Company's service.

When sixteen years old he entered Trinity College, Cambridge, to pursue mathematics, gaining, in 1825, a Trinity scholarship, and coming out fourth wrangler in 1827. He was prevented from taking his M. A. degree, or from obtaining a Fellowship, by his conscientious objection to signing the theological tests then required at Cambridge. Jevons says: "A strong repugnance to any sectarian restraints upon the freedom of opinion was one of De Morgan's most marked characteristics throughout life."

At the age of twenty-two he became professor of mathematics in University College, London. As a teacher, De Morgan was particularly gifted. A voluminous writer on mathematics, he contributed essentially to those expansions of the fundamental conceptions which have rendered possible the new algebras, such as Quaternions and the Ausdehnungslehre, and have generalized the whole idea of a mathematical algorithm or calculus.

But it is his logical work that will give De Morgan his most lasting fame. Here he stands alongside of his immortal contem-

porary, Boole. The eternally memorable year in the history of Logic was 1847, in which George Boole issued "The Mathematical Analysis of Logic, being an Essay toward a Calculus of Deductive Reasoning," and De Morgan published his principal treatise, called "Formal Logic; or, the Calculus of Inference, Necessary and Probable."

It is much to be regretted that the great memoirs produced in 1850, 1858, 1860, 1863, by De Morgan, are so comparatively inaccessible in the "Cambridge Philosophical Transactions," for it seems impossible to adequately convey in short space their value to logicians. Certainly, his "Syllabus of a Proposed System of Logic," published in 1860, however important, cannot be taken to replace even the papers of 1850 and 1858, in which the same matters appear, and lacks altogether the charm of the memoirs.

De Morgan had met in his life an unusual amount of bad logic. His great combination of logical with mathematical learning, and his prominent position in London, the great metropolis, made him the man to whom resorted all Circle-Squarers, Angle-Trisectors, Perpetual-Motionists, etc. Adding this curious experience to his great bibliographical knowledge of what had been attempted in that way in the past, he formed a large book, called "A Budget of Paradoxes," which is one of the most interesting treatises ever written on what may be called extended fallacies.

From the broad field of his published writings it is our intention only to select, first, a few points for special mention, and then state some ideas on the general way in which he has influenced for good the world of thought.

De Morgan first gave that thorough treatment of contrary, negative, or contradictory terms which has since been so much praised. Bain says: "According to the true view of contrariety, as given by De Morgan, the negative is a remainder, gained by the subtraction of the positive from the universe; the negative of X is $U-X$, and may be symbolized by a distinct mark, x ; whence X and x are the opposites under a given universe; not- X is x , and not- x is X ." It is just in reference to this point as to the term or name that De Morgan says: "Next it is clear that a name excludes as well as includes: every object of thought is related to *man*, for instance, as either *in* the name or *out*. The logician has always excluded the privative name, *not-man*, for instance, as all but use-

less: a certain practice of his own really makes it so. For he will have no *universe*—or total sphere of thought—except the whole universe of possible existence; or, at narrowest, the whole universe of objective reality.

“He forgets that, more often than not, the universe of the existing topic of thought is limited. We are talking of animals, for example, and nothing else; then *not-man* becomes so definitively significative that we have a separate name for it, *brute*.

“Logic ought to give us that command of thought which will prevent our mental vision from being obstructed by the casual absence of a name.”

Of the separation of logic and mathematic our author says: “The effect has been unfortunate. . . . The sciences of which we speak may be considered either as disciplines of the mind, or as instruments in the investigation of nature and the advancement of the arts.

“In the former point of view their object is to strengthen the power of logical deduction by frequent examples; to give a view of the difference between reasoning on probable premises and on certain ones by the construction of a body of results which in no case involve any of the uncertainty arising from the previous introduction of that which may be false; to establish confidence in abstract reasoning by the exhibition of processes whose results may be verified in many ways; to help in enabling the student to acquire correct notions of generalization; to give caution in receiving that which at first sight appears good reasoning; to instil a correct estimate of the powers of the mind by pointing out the enormous extent of the consequences which may be developed out of a few of its most fundamental notions; and to give the luxury of pursuing a study in which self-interest cannot lay down premises nor deduce conclusions.

“As instruments in the investigation of nature and the advancement of the arts it is the object of these two sciences to find out truth in every matter in which nature is to be investigated, or her powers and those of the mind to be applied to the physical progress of the human race, or their advancement in the knowledge of the material creation.”

Though fond of laughing at metaphysics, De Morgan did not see that it could be entirely gotten rid of.

"Right or wrong," he says, "human beings are made to be metaphysicians, children most of all, uneducated persons more than educated. We know all about *can* and *cannot* from our cradles; we never feel the same assurance about *is* and *is not*. A philosopher, in a dark age, may determine to set out with a knowledge of the naturally possible and impossible; but not even a philosopher ever pretended to set out with a knowledge of the existent and non-existent."

In our author's system the order of the premises is changed, the Minor Premise being written first—*e. g.* :

Vinegar is an acid.

All acids contain hydrogen.

∴ Vinegar contains hydrogen.

Thus we see instantly what before was considered very wise in Lord Bacon to have observed, that the form of the syllogism resembles the mathematical axiom—things equal to the same thing are equal to each other—for by this arrangement we bring the middle terms right together, and see just how the extremes are brought into connection through their relation to the same middle term. And so we see immediately why every Syllogism must have three, and only three, Terms—the Minor, Major, and Middle. If there be four terms, either in form or in fact (from the ambiguity of either of them), the two terms of the conclusion will not have been compared with one Middle Term, and the regular conclusion does not follow.

We have in the same way the following canons for testing the validity of Syllogisms :

1. If the Minor and Major Terms, each being compared with the same third or Middle Term, both agree with it, they agree with each other.

This underlies all Affirmative Conclusions.

2. If the Minor and Major Terms, both being compared with the same third term, one agrees and the other disagrees with it, they disagree with each other. This is the foundation of negative conclusions.

Further, Aristotle and all the old logicians said that the whole of the middle term must be taken in at least one of the premises.

As they put it, the middle term must be distributed at least

once in the premises, otherwise the minor term may be compared with one part and the major with another part of it.

From

Some men are poets,
Some men are Indians,

nothing follows. But the Aristotelians were too broad in their generalization, as De Morgan clearly showed in his doctrine of Plurative Judgments.

For example, if we have given the premises,

Most men are uneducated,
Most men are superstitious,

according to Aristotle we are not warranted in drawing any conclusion; for the middle term is men, and in neither premise is anything said about all men. But, in point of fact, we can draw the perfectly valid conclusion,

Some uneducated men are superstitious.

Again, Aristotle is contradicted by numerically definite judgments. In these there is inference when the quantities of the middle term *in the two premises together* exceed the whole quantity of that term. Lambert first thought of this principle. De Morgan, without any knowledge of Lambert, reconceived it and extended its use.

Suppose we grant the premises,

Two thirds of all human beings are women. The number of married women is never greater than the *total* number of men.

It follows that half the entire number of women are single.

Still, easy and certain as such reasoning is, it looks very like an example of how difficult it is, to a logician trained only in the traditional logic, that in a Princeton "Manual of Logic" the only numerically definite syllogism given was erroneous, and stood so for years. I stated this to the author, and in the latest stereotyped edition it has been changed. The Syllogism he gave was as follows:

"60 out of every 100 are unreflecting.

"60 out of every 100 are restless.

"Therefore, 20 out of every 100 restless persons are unreflecting."

After pointing out to him the fault in what he had been teaching for years, the following has been substituted :

"60 out of this 100 are unreflecting.

"60 out of this 100 are restless.

" $\therefore$ 20 restless persons are unreflecting."

Another point on which Aristotle and the old logicians laid great stress was what they termed the Figure of a syllogism. This they treated cumbrously and at indefinite length. De Morgan has given a much more concise, but at the same time much clearer, exposition of it. The Figure of a syllogism depends upon the situation of the middle term in the premises.

There are four figures. In the first figure the middle term is the subject of the major and predicate of the minor. In the second, the middle term is the predicate of both, and in the third the subject of both. The fourth occurs when the middle term is made the predicate of the major and subject of the minor premise.

De Morgan represents the subject of the conclusion—that is, the minor term—by x , the middle term by y , and the predicate of the conclusion—that is, the major term—by z , and says :

"A Syllogism is the deduction of a relation between two terms from the relation of each term to a third. The first figure of the logicians is that of *direct transition*— x related to z through x related to y and y to z .

"The fourth figure is that of *inverted transition*— x related to z through z to y and y to x .

"The second figure is that of *reference to* (the middle term)— x related to z through x to y and z to y .

"The third figure is that of *reference from* (the middle term)— x related to z through y to x and y to z . Thus, when the notion of figure is taken into account, its force and meaning are best seen by stating the combination of relation in the different figures.

"So when we say

"Kings are men,

"All men are mortals,

"Therefore, kings are mortals,

we are saying that kings being a species of men, men in turn being a species of mortal beings, therefore, kings are a species of mortals; and we are speaking in the first figure, for when we say that a species of a species is a species of the genus, we

compare the minor with the major by the relation which the minor stands in to the middle, and the middle to the major.

“When we say

“Common salt contains no hydrogen,

“All acids contain hydrogen,

“ $\therefore$ Common salt is not an acid,

we use the second figure, and compare both major and minor with the middle.

“This is equivalent to saying species and genus of the same are species and genus of one another.

“Again, if we say

“No tribes are without religion,

“Some tribes are cannibals,

“ $\therefore$ Some cannibals are not without religion,

we use the third figure and compare the middle term with both major and minor.

“The fourth figure is simply the first with the concluding relation inverted.”

Here we see stress laid upon the consideration of relations, and, after all, we feel sure that De Morgan's greatest work was connected with his development of the Logic of Relatives, independently discovered by Leslie Ellis after reading Boole's "Laws of Thought." One of De Morgan's last memoirs, in the tenth volume of the "Cambridge Transactions" was on the Logic of Relations, which is, in the mathematical sense, a far-reaching generalization of the old logic. In our modern mathematics everything is generalized as far as possible. Thus the notion of imaginary magnitude in analysis, and of imaginary space in geometry, has become fundamental and all-embracing, including the real as a special case. Every study of a generalization or extension gives additional power over the particular. We need to go beyond and look back from an elevation.

Any first-rate mathematician working in logic would attempt to generalize, and, in fact, Boole generalized, the scholastic logic in a manner entirely different from De Morgan. In the "Vierteljahrsschrift für wissenschaftliche Philosophie," page 250, the celebrated critic, A. Riehl, says: "Das allgemeine, den aristotelischen Syllogismus als speciellen Fall einer speciellen Methode umfassende Problem des Schliessens hat Boole nicht nur gestellt,

sondern auch gelöst. Niemand, der sich gegenwärtig oder künftig mit Reform der Logik beschäftigt, darf am Werke Boole's vorbeigehen." Boole is obtaining world-wide homage. Herbert Spencer says: "In the work by Professor Boole, 'Investigation of the Laws of Thought,' the application to Logic of methods like those of mathematics constitutes another step far greater in originality and in importance than any taken since Aristotle."

But let us not forget that De Morgan's generalization, though different in kind and direction, has done, perhaps, as much toward breaking away the old incrustation that covered the logic germs.

In his view of the subject, the purely formal proposition with judgment, wholly void of matter, is seen in "There is the probability x that X is in the relation L to Y ." The syllogism is the determination of the relation which exists between two objects of thought by means of the relation in which each of them stands to some third object which is the middle term.

The pure form of the syllogism, when its premises are absolutely asserted, is as follows: X is in the relation L to Y , Y is in the relation M to Z ; therefore X is in the relation " L of M ," compounded of L and M , to Z . In ordinary logic, which admits only the relation of identity, the actual composition of the relation is made by our consciousness of its *transitive* character. The requisites of the copular relation, in the system of ordinary syllogism, are *convertibility* and *transitiveness*.

Any relation which possesses these qualities may take the place of "is" in the common syllogism without impeachment of its validity.

A relation is transitive when, being compounded with itself, it reproduces itself; that is, L is transitive when every L of L is L . For example, "brother." Thus, from the transitiveness of the connecting relation in ordinary syllogism, A is B and B is C gives A is C , since, from the *convertibility* of the terms, A is B gives B is A . Here at last we have broken away from that paltry narrowness which sickens us with the assertion that our minds in pure thinking can use nothing but the relation of identity—the Jevons sophism that thought cannot move because all thinking is the substitution of identicals.

So we see that in logic, as in mathematics, we may develop a

whole system of theorems about symbols which are to be used in a given manner; and then to make this whole system true of a desired relation we have only to show that the relation fulfils the one or two fundamental principles of the system. De Morgan treated of convertible and inconvertible relatives, repeating relatives, non-repeating relatives, transitive and intransitive relatives, and inaugurated a general system.

On three out of his four pairs of simple propositions three separate algebras of logic have been founded.

Resurrected and revived, Logic has joined the ranks of the on-marching sciences.

ON HEGEL'S IDEA OF THE NATURE AND SANCTION OF LAW.

BY WALTER B. WINES.

There can be no doubt as to the necessity for the acceptance of the inevitable. To accomplish the possible, and to refrain from attempting the impossible, are equally wise. From this admitted truth, as a major premise, with a minor premise supplied in each particular case, a practical age has constructed a prudential syllogism whose conclusion is that to avoid vain seeking after empty knowledge and useless inquiry after that which knowledge cannot compass is not less commendable than to know all things knowable. It should not, however, be forgotten that prudence, while often the soundest worldly wisdom, may sometimes be contemptible meanness. To the palace built by philosophy, prudence sustains the relation of a cellar to a house: fundamental, useful, even necessary, yet not forming a part of its symmetry, and far beneath the apartments above, illumined by the sunlight, and through which sweeps the pure upper air.

The maxim, "Seek not to know what you cannot know," commends itself in many respects. The proposition that it is possible to know only what is capable of knowledge calls for no argument. It is not its statement, but its application, that is deleterious. The